Math 331   **<u>Final Exam - Written Portion</u>**   **Spring 2020**

In this portion of the exam there are 9 pages, including this one. There are 6 problems and an Honesty Statement to be copied.

Instructions:

- The textbook and notes created by you or found on our Moodle course are allowed to be used during the exam.

- A calculator is allowed to be used during the exam.

- Any other outside sources are **not allowed** to be used during the exam.

- You must explain how you arrived at your answers, and show your algebraic calculations.

- Please leave fractions and square roots in your answers – do not give decimal expansions.

Question	Points	Score
1	15	
2	15	
3	20	
4	20	
5	15	
6	15	
Total:	100	

Please read this paragraph carefully. To assure us that you have read it and fully understand it, **please rewrite it in the space below**:

"For this exam, I pledge that I have not received, not given, nor will I give or receive, any assistance to another student taking this exam, including discussing the exam with students in another section of the course. I will not plagiarize someone else's work and turn it in as my own. I will not disclose any exam content before, during, or after I have taken the exam. This includes taking screenshots or pictures of the questions on this exam and sharing them on any websites. I understand that acts of academic dishonesty may be penalized according to the University of Massachusetts Amherst Academic Honesty Guidelines, including receiving a failing grade for the course. I recognize that I am responsible for understanding the provisions of the University of Massachusetts Amherst Academic Honesty Policy and Procedures."

Signature: _____

1. Consider the following system

$$x' = A x, \quad \text{with} \quad A = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \quad \text{and} \quad x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

(a) (9 points) Find the eigenvalues and eigenvectors of the matrix A

(b) (6 points) Solve the initial value problem

2. (15 points) Solve the initial value problem

$$y'' + 7y' + 12y = \delta(t - 2) \quad \text{with initial conditions} \quad y(0) = 0 \quad \text{and} \quad y'(0) = 1$$

using the Laplace transform and make a graph of the solution.

3.

(a) (12 points) Use Laplace transforms to solve the following initial value problem

$$y'' + 4y' + 16y = u_2(t) - u_4(t) \quad \text{with initial conditions } y(0) = 1 \text{ and } y'(0) = 0.$$

(b) (8 points) Find $\lim\limits_{t \to \infty} y(t)$

4. This question has three parts over two pages.

  (a) (8 points) Find the general solution to the differential equation

  $$y'' + 5y' + 6y = t.$$

  (b) (5 points) Find the general solution to the differential equation

  $$y'' + 5y' + 6y = e^{2t}.$$

  (You may use your work from part (a)).

(c) (7 points) Find the particular solution to the initial value problem

$$y'' + 5y' + 6y = t + e^{2t}, \quad y(0) = \frac{41}{45}, \ y'(0) = \frac{19}{15}$$

(You may use your work from parts (a) and (b)).

5. (15 points) Find the General Solution to the Linear System of Differential Equations

$$x' = 4x + 13y$$
$$y' = -2x + 2y$$

6. Find the general solution of the following second order homogeneous linear differential equations:

   (a) (8 points) y"(t) - 6y'(t) + 25y(t) = 0

   (b) (7 points) y"(t) - 6y'(t) + 9y(t) = 0